Math Placement Test Practice: Elementary Algebra Practice

Quick Refresher for Selected Topics

Purpose: *This refresher is not intended to teach new concepts. Its purpose is rather to remind students of concepts once learned in a previous college or high school course.*

Operations with Integers

- **Absolute value** is the distance from the origin. So, the absolute value is a non-negative number.
- The order of mathematical operations is not left to right, but instead it follows a hierarchy described by the acronym **PEMDAS**. The order of operations is…
    - **"P"** represents operations within parentheses or grouping symbols
    - **"E"** represents exponentiation operations
    - **"MD"** represents the operations of multiplication and division. Both multiplication and division are operations at same level and are therefore done left-to-right in the order that they appear
    - **"AS"** represents operations of addition and subtraction. Addition and subtraction are operations at same level and are therefore done left-to-right in the order that they appear.
- An **exponent** tells how many times the base is multiplied by itself. For example, $\mathbf{2^3 = 2 \cdot 2 \cdot 2}$.
- Remember that a number directly preceding an open parenthesis means the quantity prior is multiplied by the quantity within, for example $\mathbf{5(7) = 5 \cdot 7}$
- **Subtraction** is just a **special case of addition,** just adding a negative, for example $\mathbf{3 - 2 = 3 + (-2)}$
- **Division** is just a **special case of multiplication,** just multiplying by the inverse of the divisor, for example $3 \div 2 = 3 \cdot \frac{1}{2}$

Operations with Fractions

- The **denominator** of a fraction is the *denomination* of a whole (how many equal parts are in a whole.) The **numerator** is the number of those parts in the fraction. So $\frac{5}{6}$ is five slices of a pie that has been cut into six equal-sized pieces.
- An integer has a denominator of 1, because for an integer each whole has 1 part.
- **Simplest form** means the numerator and denominator of a fraction have no common factors
- The process of **multiplying fractions** involves 1) canceling any factors common to the numerators and denominators, 2) multiplying the numerators, 3) multiplying the denominators, and 4) reducing the answer to its simplest form
- The process of **dividing fractions** involves 1) inverting the fraction after the division symbol and then 2) multiplying the fraction before the division symbol by the inverted fraction afterwards

- To be added or subtracted, fractions must have identical denominators. **Common denominators** can be created by multiplying each fraction by the appropriate form of 1 in disguise. For example: $\frac{7}{18} - \frac{3}{15} = \frac{7}{18} \cdot \frac{5}{5} - \frac{3}{15} \cdot \frac{6}{6} = \frac{35}{90} - \frac{18}{90} = \frac{17}{90}$.

Percentages, Decimals and Fractions

- Any fraction with a numerator that is more than one-half of its denominator is greater than one-half. Likewise, any fraction with a numerator that is less than one-half of its denominator is less than one-half.
- Each piece of a pie cut into more equal sized pieces is smaller. So, fractions with **identical numerators** become smaller as their denominators become larger. Since the numerator represents the number of pieces of a pie, fractions with **equal denominators** are larger, if their numerators are larger.
- Since the fraction bar is equivalent to a division symbol, a fraction can be **converted to a decimal** by dividing the numerator by its denominator
- A decimal can be **converted to a fraction** by 1) putting the decimal over 1, 2) multiplying the numerator and denominator by a multiple of a 10 that eliminates the decimal, and 3) simplifying the resulting fraction
- A percent is defined as a ratio out of 100, $\frac{\%}{100} = \frac{part}{whole}$
- Since a **decimal and its percent are multiples of 100** of each other, the percent can be found by multiplying the decimal by 100. The decimal can be found by diving its percent by 100. The percent is numerically larger than its decimal form.
- When writing algebraic equations, the word "is" means equals. The word "of" often signifies multiplication.

Geometric Calculations

- **Perimeter** is distance along the outer edges of a figure. Circumference is the distance around a circle. Perimeter and circumference can be thought of as the length of a fence around the figure. Since they are lengths, perimeter and circumference have **dimensions of length**.
- **Area** is the amount of surface within the two-dimensional figure. It has **dimensions of length squared**
- Since a triangle is equivalent to a parallelogram divided by two, its area is one-half of that of the parallelogram, so for a triangle $A = \frac{1}{2}bh$.
- Often students confuse the equations for area and circumference of a circle. One way of preventing this confusion is thinking about the dimensions of area and circumference. Since area has units of length squared, the radius must be squared, $A = \pi r^2$. Since circumference has units of length, the radius must be to the first power, $C = 2\pi r$.

Operations with Algebraic Expressions

- Do not confuse addition or subtraction, such as $(3x^2 - 5x + 7) - (4x^2 + x + 1)$, with multiplication, such as $(3x^2 - 5x + 7)(4x^2 + x + 1)$
- **Squaring** a polynomial creates additional **middle terms**, such as $(2x + 5)^2 = (2x + 5)(2x + 5) = (4x^2 + 20x + 25)$. An exponent cannot not be simply distributed through the parentheses when there is addition or subtraction within the parentheses
- When **adding or subtracting like terms**, the **exponents** of the variables **do not change**
- When multiplying terms, the exponents for variables may change
- Follow order of operations when performing arithmetic with polynomials

Operations with Exponents and Roots

- Only **like roots** can be combined by addition or multiplication, such as $\sqrt{7} + 3\sqrt{7} = 4\sqrt{7}$
- Scientific notation has the form of $b \; x \; 10^n$, where $1 \leq b < 10$ and n is an integer
- The **exponents rules** are…
    - $x^0 = 1$
    - $x^{-n} = \frac{1}{x^n}$
    - $x^a x^b = x^{a+b}$
    - $\frac{x^a}{x^b} = x^{a-b}$
    - $(kx)^a = k^a x^a$
    - $\left(\frac{x^a}{y^b}\right)^c = \frac{x^{ac}}{y^{bc}}$

Factoring

- Always try to **remove a greatest common factor** first
- If the highest order term is negative, **remove a factor of** $\mathbf{-1}$
- If **four terms**, try factoring by **grouping**
- If **two terms**, try factoring by **difference of squares**
- If **three term** and the **leading coefficient is 1**, find the factors of the constant whose sum equals the coefficient of the middle term
- If **three terms** and leading term and constant are **both perfect squares**, first try using their square roots.
- Remember all factoring can be **checked by multiplying the factors**

Operations with Rational Expressions

- Leave the final **answer in factored form**
- When **multiplying**, factor the numerators and denominators, cancel, and then multiply
- When **dividing**, invert the fraction after the division symbol, replace the division symbol with multiplication, and follow the steps for multiplication

- When simplifying **complex fractions**, flip the bottom fraction and multiply (This assumes the original denominator of complex fraction is a single fraction)
- When **adding or subtracting**, factor the denominators, find the common denominator, make all denominators common by multiplying by one in disguise, and combine any like terms in the numerators

Solving Equations and Inequalities

- **Answers can be checked** by substituting the result into each side of the equation or inequality and determining if the equality or inequality is true.
- When solving problems involving fractions, **make all non-fractions into fractions** by placing them over 1. Then eliminate the denominators by either making them all common or multiplying each term by the common denominator.
- If an equation contains polynomials in the denominators, **factor the polynomials** to find the common denominator.

Solving Systems of Linear Equations

- Each equation in the system represents a line. The two lines can **intersect**, be **parallel**, or be the **same line**. The solution set is where the lines coincide. Intersecting lines have **one solution** at the coordinates of intersection. Parallel lines have **no solution**, since they never cross. Identical lines have **infinite solutions** since the lines coincide at every point on the line.
- The **substitution method** is easier to use if in one equation a variable has a coefficient of 1 or -1, because no fractions are then created when solving for that variable in that equation
- The **elimination method** is the second method

Linear Functions and Their Graphs

- Key equations:
  - **Slope**: $m = \frac{y_2 - y_1}{x_2 - x_1}$, for points (x_1, y_1) and (x_2, y_2)
  - **Slope-intercept**: $y = mx + b$, where m is the slope and $(0, b)$ is the y-intercept
  - **Point-slope:** $y - y_1 = m(x - x_1)$, where m is the slope and (x_1, y_1) is a point on the line
- **Intercepts** occur where the line crosses the axis, so the opposite coordinate of an intercept is zero. This means that the y-coordinate of the x-intercept is zero and the x-coordinate of the y-intercept is zero.
- To put an equation of a line in **slope-intercept form, solve for y**
- To find the **equation of a line**, find the slope either from two points or from a parallel or perpendicular line, **substitute the slope and a point into the point-slope form**, and solve for y
- **Parallel lines** have the **same slope**. **Perpendicular lines** have slopes that are the **negative reciprocal** of each other.

Applications

- Key words for operations in algebraic expressions and equations are…
    - **Addition:** sum, more than, added to, …
    - **Subtraction:** difference, less than, subtracted from, …
    - **Multiplication:** of, times, product, …
    - **Division:** quotient, ratio, …
    - **Equal sign:** is, equals, amounts to, …
- Be aware that **order matters in subtraction and division**. The order is sometimes in the mathematical phrase in the opposite order of the written phrase. For instance, a number less than 3 is written algebraically as $3 - x$.
- When solving application problems**, define the meaning of your variables** in writing
- Most often **the quantity you know nothing about** should be assigned a **variable**. For example, for the phrase "the length is five less than twice the width," nothing is known about the width and the length is defined in terms of the width. So, the width should be assigned variable, say w. Then the length would equal the expression $2w - 5$.
- Often it is helpful to **organize** the information from the problem in a **diagram or table**. Also, **list** the **knowns** and **unknowns**.
- **Pythagorean's Theorem**, $a^2 + b^2 = c^2$, applies to a right triangle where a and b are the lengths of the legs and c is the length of the hypotenuse, which is across from the right angle
- Problems involving travel often make use of $\boldsymbol{d = rt}$, distance = (speed)(time)